2mic

NASA TECHNICAL TRANSLATION NASA TT F-15,659

OUTLINE OF RADIATIVE TRANSFER THEORIES APPLICABLE TO PROPAGATION IN THE SEA

L. Prieur and A. Morel

Translation of: "Apercu sur les Theories du Transfert Radiatif Applicables a la Propagation dans la Mer," in: Optics of the Sea (Interface and In-Water Transmission and Imaging), Advisory Group for Aerospace Research and Development, AGARD LS-61, Neuilly sur Seine, France, August 1973, 25 pages

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 JULY 1974

STANDARD TITLE PAGE

1. Report No. NASA TT F-15,659	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Outline of Radiative Transfer Theories Applicable to Propagation in the Sea		5. Report Date JULY 1974
		6. Performing Organization Code
7. Author(s) L. Prieur and A. Morel		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Techtran Corporation P.O. Box 729 Glen Burnie, Maryland 21061		11. Contract or Grant No. NASw-2485
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address NATIONAL AERONAUTICS AND SPACE ADMINISTRATION WASHINGTON, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of: "Apercu sur les Theories du Transfert Radiatif Applicables a la Propagation dans la Mer," in: Optics of the Sea (Interface and In-Water Transmission and Imaging), Advisory Group for Aerospace Research and Development, AGARD LS-61, Neuilly sur Seine, France, August 1973, 25 pages

16. Abstract

A survey is made of current theories of radiation transfer which may be applied to the propagation of electromagnetic waves in seawater. The physical factors providing optical characterization of seawater are reviewed, and the various methods of caluculating them are discussed in brief. An evaluation is made of the methods currently employed and the advantages and disadvantages of each method are presented.

17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified-Unlimited		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 47	22. Price 5.50

NASA-HQ

OUTLINE OF RADIATIVE TRANSFER THEORIES APPLICABLE TO PROPAGATION IN THE SEA[1]

L. Prieur and A. Morel[2]

Introduction /1.3.1*

The theories currently in existence were developed during research relating primarily to electromagnetic radiation deriving from planets or stars, and more recently to neutron scattering. The radiative transfer problems may in all instances be stated in mathematical terms, in the form of a transfer equation. Solution of this equation presents no difficulty whatever when the medium in which the particles (photons, neutrons, etc.) are propagated is not a diffusing medium, even if it does itself contain sources of these same particles. The situation is entirely different if redistribution throughout space due to scattering occurs. It is then quite often assumed for the sake of simplification that the scattering is isotropic or that the absorption is slight and may be disregarded. Such approximations are generally justifiable for the study of neutron scattering or in astronomy.

out

Since oceanography began its development at a relatively recent date, many fewer theoretical studies have been devoted to the propagation of visible electromagnetic waves in the sea. On the other hand, a considerable body of experimental data is available, and a theory, of necessity a fairly complex one in the case of seawater, might be of a certain value. As a matter of fact, it is necessary to resort to one to be able to make forecasts in widely varying fields. For example, we wish to know and forecast on the basis of the measurable characteristics of water, the distribution of energy available for photosynthesis, a fundamentally important element in life. We wish to determine how the solar radiation absorbed by oceans participates in the radiation balance of the planet. A theory is further necessary for

[1]Part of this bibliographic discussion is covered by contract DRME 72-555.
[2]Physical Oceanography Laboratory, Villefranche Oceanographic Research Center, 06230 Villefranche-sur-Mer, France.
*Numbers in the margin indicate pagination in the foreign text.

forecasting visibility, for selecting suitable equipment for photogrammetry, for interpreting aerial photographs of the surface of the sea, etc.

The I. Description of The Problem and Factors Involved in Its Solution

Before reviewing the principal radiative transfer theories it is appropriate to recapitulate the physical factors providing optimum characterization of the medium and the photometric values permitting description of penetration of the sea by electromagnetic waves. These basic concepts are discussed in detail in the classic works and are dealt with in papers on the same subject (for example, [73, 38, 22, 36, 53]).

I.1. Optical Characteristics

The attenuation of a collimated beam is characterized by (Napierian) attenuation coefficient $c(m^{-1})$. Decrease in the flux of such a beam is due to two phenomena: absorption, Napierian coefficient $a(m^{-1})$, and diffusion redistributing the energy throughout space in accordance with a scattering indicatrix (Napierian coefficient $\beta(\theta)$ $(m^{-1}ster^{-1})$). The integral over all of solid angle $\beta(\theta)$ is termed the total scattering coefficient $b(m^{-1})$. These coefficients, which are defined for a collimated beam, as a matter of fact describe the elementary events capable of occurring to a photon. The law of conservation of energy necessitates the relations

$$b = \int_{4\pi} \beta(\theta) d\Omega = 2\pi \int_0^{\pi} \beta(\theta) \sin\theta \, d\theta, \tag{1}$$

$$c = a + b. \tag{2}$$

The scattering indicatrix of seawater is very asymmetric and very "pointed" toward the front [53], and absorption by such waters is not negligible even when they are considered to be transparent. Because of these two facts, any theory constructed for an isotropically scattering or slightly asymmetric medium, or for a medium exhibiting negligible absorption, is not realistic enough to be applied to the ocean. These are important considerations, for they lead to elimination (or modification) of the majority of methods of solving the radiation transfer equation which have been established in astronomy or for neutron scattering.

On the other hand, the form of the scattering indicatrix undergoes but little variation for the various waters of the sea, except perhaps at very small angles and in the rear portion [53, 51, 52]. There is consequently a nearly constant relationship between total scattering coefficient b and angular coefficient $\beta(\theta)$. This results in introduction into calculations of the scattering indicatrix in a normalized form and in allowance for the absolute value only in parameter b. Ratio b/c, which to some extent characterizes the probability that a photon will be scattered along a path of optical length τ[1], assumes great importance in this case. As a matter of fact, the probability that this photon will be scattered in a given direction θ is then proportional to b/c whatever the type of water encountered. The importance of this ratio for description of penetration of light into turbid media has been ascertained chiefly by way of experiment, by V. A. Timofeyeva [89], for example. In mathematical form, as it is described in what follows, this ratio appears explicitly in the transfer equations. /I.3.2

I.2. Values Characterizing the Distribution of Luminous Energy

The basic photometric value is luminance L(P, D), which is the flux received, normally in P, by an elementary surface dS within an elementary solid angle $d\omega$ enclosing direction D. This quantity describes the light field completely, at all points in space and for all directions. However, it is also the quantity which it is the most complex to obtain both in theory and by way of experiment. The luminance integrals are often considered over a half-space in calculations, in order to obtain the illumination of a plane, or over the space in its entirety in order to obtain the scalar illumination:

The illumination of a plane is (see Figure 1):

$$E = \int_{2\pi} L.\cos\theta \, d\omega' \qquad (3)$$

The scalar illumination is:

$$E_o = \int_{4\pi} L \, d\omega \qquad (4)$$

[1] $\tau = cz$, in which z is the actual distance expressed in meters. Hence τ is a dimensionless value and for this reason the terms optical depth and optical length are inappropriate. They are used here in imitation of English usage.

I.3. Influence of Wavelength

Radiation is assumed in the majority of theoretical calculations to be monochromatic. This is justifiable, since the physical scattering phenomena (fluorescence, Raman effect) are very often negligible in study of the propagation of solar or artificial radiation in the sea. In addition, such an assumption is necessary because the optical characteristics depend on the wavelength. However, it must be remembered that the form of the indicatrix does not depend on it too much in the first approximation [54] [51b]. Thus in measurement in which a photometric value depends only on ratio b/c, a change in wavelength is equivalent simply to change in the value of this ratio.

I.4. Boundary Conditions

The optical characteristics presented in the foregoing define the medium itself in its elementary interactions with electromagnetic waves. In order fully to define the problem it is appropriate to assign the boundary conditions, which to some extent represent the initial data. A certain distribution of luminances over the surface of the sea is assumed and the distribution in depth is sought, or the emission indicatrix of a submerged lamp is adopted, and so forth. These conditions are independent of the optical characteristics of the medium itself and relate only to the geometric distribution of the luminances within the limits of the medium. Generally speaking the choice of coordinate system will depend upon their symmetry, as will also choice of the method to be employed to gain knowledge of the distribution of the luminances in depth.

Before the problem is solved in theory, we are forced to render the data ideal so that they will exhibit a simple mathematical form. It is assumed for study of penetration by daylight that the surface of the sea is illuminated uniformly in all directions, or only by the Sun, that is, in a single direction. The bottom of the sea is assumed to be of infinite depth or that it has a zero or uniform reflection coefficient. For penetration by artificial light the source is collimated or obeys Lambert's law and further is assumed to be a point source.

I.5. General Considerations

A detailed survey of radiation transfer has been made in several more or less recent publications. Generally speaking their authors have devoted themselves to study of the problems deriving from astronomy, and thus none of the papers deals with the particular case of seawater. The basic principles of certain theories presented nevertheless remain valid, only the methods of solution undergoing change in details. The most complete work on the various current theories unquestionably appears to be that of V. V. Sobolev [85]. Other fundamental works discuss certain points in greater detail, such as those of S. Chandrasekhar [16], V. Kurganov [40], and R. W. Preisendorfer [71], [72].

In our presentation of the various methods of solving the radiation transfer problems (II) we have endeavored to single out the physical concepts on which the methods are based, whenever such concepts exist. In addition, the only theories presented are those which can bring into play an asymmetric indicatrix corresponding to the case of seawater. We subsequently consider (III) several specific problems already studied by various authors. They have been rendered ideal enough to permit application of the previously described theories, but not excessively so, so that the results will satisfactorily cover the actual physical conditions.

II. Methods

There are indeed very many methods, which are used in more or less different forms by various authors. It is often difficult to distinguish one from the other. Their classification into different types, although necessary for the sake of clarity, may appear to be arbitrary because of their overlapping. /1.3.3
However, they are here classified into five broad categories each of which involves different mathematical or physical considerations.

- The first category includes the methods which resort to mathematical expedients in order to solve the transfer equation with a minimum of approximations.

- The second category embraces the natural methods permitting calculation of the various orders of scattering in sequence.

- In the third category resort is no longer made to the simple process of attenuation and scattering but to diffuse reflection and transmission operators corresponding to a layer of the medium.

- In the fourth category the real propagation of a photon is simulated.

- Lastly, there have been included in the fifth category the theories which assume a random variation of the scattering process in time, while in the preceding methods the phenomena are generally assumed to be independent of time.

II.1. Mathematical Methods

All these methods make use of the transfer equation, which is generally obtained on the basis of local conservation of energy. However, no consideration is given to the physical import of the calculations performed in solution of the equation.

II.1.1. Transfer Equation

It is assumed for the sake of simplicity that the phenomena are stationary, and that the radiation is monochromatic. The energy balance is expressed at each point P of the medium, and for each direction D, for an element of volume $d\ell \cdot dS$ (see Figure 2).

The difference between the flux entering this volume and that leaving it, depending on direction D, is:

$$\frac{dL\,(P,D)}{d\ell} \cdot dS \cdot d\ell \cdot d\omega \tag{5}$$

- The attenuation loss in direction D is, in accordance with the definition of c,

$$-\,c(P)d\ell \cdot L(P,D) \cdot dS\,d\omega \tag{6}$$

- The scattering gain in direction D in energy penetrating element of volume dV and coming from direction D' is (see the definition of β):

$$+\left[\int_{4\pi} \beta\,(P, D, D') \cdot L\,(P, D')\,d\omega'\right] \underbrace{d\ell \cdot dS}_{dV} \cdot d\omega \tag{7}$$

The law of conservation of energy requires that expression (5) and the sum of expressions (6) and (7) be equal,

$$\frac{dL\,(P, D)}{d\ell} = -\,c(P)\,L(P,D) + \int_{4\pi} \beta(P, D, D') \cdot L(P, D')\,d\omega', \tag{8}$$

which is the transfer equation under stationary conditions (independent of time) and with no internal source.

All the methods in this group assume that the optical characteristics are independent of the space variable, which here is symbolized by P. In addition, it is convenient to normalize the scattering indicatrix by its integral, and thus to show the total scattering coefficient explicitly. Equation (6) becomes

$$\frac{1}{c} \cdot \frac{dL(P, D)}{d\ell} = - L(P, D) + L^{*} (P, D). \tag{9}$$

in which, by definition,

$$L^{*} (P, D) = \frac{b}{c} \cdot \int_{4\pi} L (P, D') \beta_n (D, D') d\omega' , \tag{10}$$

and

$$\int_{4\pi} \beta_n (D, D') d\omega' = 1 \text{ quelque soit } D. \tag{11}$$

whatever the value of D may be.

Function L* is termed the radiation transfer source function and sometimes the "path function" [22].

Integration over solid angle ω leads to a simple relation:

$$\text{Div } E = - a E_o \tag{11b}$$

which is another form expressing the local conservation of energy.

In the case of penetration by daylight the surface illumination does not depend on horizontal coordinates, since it is assumed to be constant over the entire surface. Then only depth z as a variable and direction D remain. When optical depth τ = cz and angular coordinates (see Figure 3) are introduced, the transfer equation is written as follows: /1.3.4

$$\mu \frac{dL (\tau, \mu, \phi)}{d\tau} = - L (\tau, \mu, \phi) + \frac{b}{c} \int_{-1}^{+1} \int_{0}^{2\pi} L (\tau, \mu', \phi') \beta_n (\alpha) d\mu' d\phi' \tag{12}$$

with

$$\cos \alpha = \mu\mu' + \sqrt{(1 - \mu^2) (1 - \mu'^2)} \cos \phi \tag{13}$$

$$\mu = \cos \theta \tag{14}$$

This is the homogeneous transfer equation for a medium with "plane-parallel" symmetry, the latter being so called to point up the surface illumination conditions. Lastly, although this is not a restriction on the methods advanced later, a case is contemplated in which axial symmetry also exists, that is, in which azimuth angle ϕ has no effect on the luminances (this requiring the surface distribution to possess the same symmetry).

With these hypotheses the transfer equation becomes

$$\mu \frac{dL(\tau, \mu)}{d\tau} = - L(\tau, \mu) + b/_c \int_{-1}^{+1} L(\tau, \mu') \left[\int_0^{2\pi} \beta_n(\alpha)\, d\phi' \right] d\mu' \tag{15}$$

This equation is often called a "homogeneous equation". So that the problem will be fully defined, it is further appropriate to assign the boundary conditions. These conditions make it possible to establish the integration constants which appear when this differential equation is solved. This is often accomplished, however, by introducing into equation (15) a supplementary term representing the light directly transmitted without diffusion from the surface to depth τ. This term is of the form $L(o, \mu)e^{-\tau,\mu}$. The equation obtained is then said to have a second term. The boundary conditions of the problem as thus formulated must consequently then be changed [96, p. 1099], [16, p. 22].

II.1.2. Discrete Ordinate Method. Eigenfunction Method.

Equation (15) is an integral differential equation, and there is no simple analytical solution of such an equation. Source function L' is complex in structure because of the asymmetry of the indicatrix. The property of addition of spherical harmonics is also generally used in order to simplify this integral. The mathematical process, which has been described very completely by S. Chandrasekhar, is as follows:

- Function $\beta(\mu) = b/c\ \beta_n(\mu)$ is expanded into a series of Legendre polynomials

$$\beta(\mu) = \frac{1}{4\pi} \sum_{\ell=0}^{N} \omega_\ell P_\ell(\mu) \tag{16}$$

l being an integer. N is the total number of polynomials selected for approximation of the scattering indicatrix. It is found by integration over μ and by taking into account the orthogonality properties of the Legendre polynomials that ω_o, the first coefficient in the expansion, is merely ratio b/c (the zeroth order polynomial being unity).

The property of spherical harmonic addition leads to

$$\int_0^{2\pi} \beta\ (\alpha)\ d\phi' = \frac{1}{2} \sum_{\ell=0}^{N} \omega_\ell\ P_\ell\ (\mu)\ P_\ell(\mu') \tag{17}$$

put into

Once the scattering indicatrix has been put into manageable mathematical form, the principal remaining difficulty is approximation of the integral operator of equation (15). Equation (17) being taken into account, the latter becomes

$$\mu \frac{dL}{dL}\ (\tau,\ \mu) = -\ L(\tau,\ \mu) + \frac{1}{2} \sum_{\ell=0}^{N} \omega_\ell\ P_\ell\ (\mu) \left[\int_{-1}^{+1} P_\ell\ (\mu')\ L\ (\tau,\ \mu')\ d\mu'\right] \tag{18}$$

Chiefly two methods, which will be presented in greater detail later on, have been used to solve this equation:

- The discrete ordinate method developed by S. Chandrasekhar [16]. (The reader is referred to this work for the detailed calculations.)

Use is made of the Gauss squaring method, which divides the integration interval according to the zeroes of the Legendre polynomials (other squaring methods are possible and in certain cases are more precise; as a matter of fact, /1.3.5
the scattering indicatrix development selected affects the entire calculation process. For further details on other types of development the reader is referred to the work by V. V. Sobolev [85] or to that of V. Kurganov [40]. It is logical, however, to employ the Gauss squaring, since the Legendre polynomials have already been used for development of the indicatrix).

Hence interval (-1, +1) is divided according to the zeroes μ_j of the Legendre polynomial $P_{2n}(\mu)$, which are 2n in number. The squaring formula is

$$\int_{-1}^{+1} f(\mu)d\mu \approx \sum_{j=-n}^{n} a_j\ f(\mu_j), \tag{19}$$

with

$$a_j = \frac{1}{\left[\dfrac{dP_m(\mu)}{d\mu}\right]_{\mu=\mu_j}} \times \int_{-1}^{+1} \frac{P_m\ (\mu)}{\mu - \mu_j}\ d\mu\ . \tag{19b}$$

Coefficients a_j are present in the form of numerical tables.

Equation (18) is then replaced by a system of 2n simple homogeneous differential equations of the form:

$$\mu_i \frac{dL}{d\tau}\ (\tau,\ \mu_i) = -\ L\ (\tau,\ \mu_i) + \frac{1}{2} \sum_{\ell=0}^{N} \omega_\ell\ P_\ell(\mu_i) \sum_{j=-n}^{n} a_j\ P_\ell(\mu_j)\ L\ (\tau,\ \mu_j), \text{ with } i = \pm 1, \pm 2, \ldots, \pm n \tag{20}$$

the only condition set for n is:

$$4n - 1 > 2N$$

It is said that when n is selected the equation is solved in the nth approximation.

The solution of this system is the conventional one. Solutions of the form

$$L(\tau, \mu_i) = e^{+k_\alpha \tau} f(\mu_i, k_\alpha) \tag{21}$$

are sought.

Constants k_α are the roots of the "characteristic" equation of system (20):

$$1 = \frac{1}{2} \sum_{j=+n}^{n} \sum_{\ell=0}^{N} \frac{a_j}{1 + \mu_j k} \omega_\ell P_\ell(\mu_j) \xi_\ell(k). \tag{22}$$

Values ξ_l are obtained by recurrence, and it is shown that the equation is of order n in terms of k^2; thus there are 2n different roots, which are generally symmetrical.

It is to be noted that the values of k_α depend only on the values of μ_j and coefficients ω_l of the development of the scattering indicatrix.

The solution of system (20) is written as follows:

$$L(\tau, \mu_i) = \sum_{\alpha=-n}^{n} \frac{C_\alpha e^{k_\alpha \tau}}{1 + \mu_i k_\alpha} f(\mu_i, k_\alpha), \tag{23}$$

in which $f(\mu_i, k_\alpha)$ is the symbol for a function of the parameters now known. Hence the final result shows that the luminance is expressed as a function of a sum of exponents coefficients k_α of which are characteristic of the medium itself, once number n has been selected.

Constants c_α are determined by the boundary conditions of the problem assigned; for example, if the medium is infinite, the luminance should tend toward zero for indefinitely increasing τ, and so the values of c_α should be zero for positive values of k_α.

It is to be observed that the luminances are obtained only for discrete values of angle $\theta(\mu_i)$ imposed by the Legendre polynomials of order 2n.

The calculation principle remains the same if the symmetry is not axial, since development into a Fourier series is effected:

$$L(\tau, \mu, \phi) = \sum_{m=0}^{N} L^m(\tau, \mu) \cos m\phi \qquad (24)$$ /1.3.6

and the values of $L^m(\tau, \mu)$ are calculated by a method very similar to the foregoing but somewhat more complex since m figures directly in the equations.

Thus the method remains a very general one. It is very convenient, in particular when it is a matter of gaining knowledge of the influence of surface conditions on the penetration of light into a medium having a given indicatrix, since the values of k_α remain the same and only coefficients c_α change. Hence in theory one could tabulate the values of k_α and of function $f(\mu_i, k_\alpha)$ of equation (20) for the typical values of b/c, the indicatrix being constant in form. Under this assumption solution of the problem assigned would be confined to solution of equations (23) in accordance with the boundary conditions deriving from the problem.

Eigenfunction Method

This method was initially applied to radiation transfer by Case [15] and then extended to the case of nonspherical scattering [50], [18, 19]. A survey of the method is to be found in the article by S. Pahor [56].

The transfer equation is taken again in the form of [18], that is, the medium is assumed to be plane-parallel and to have axial symmetry (see Section II.1.1.). In this instance as well the general case should be treated by development into a Fourier series in accordance with the azimuth (equation 24).

Solutions of the form

$$L(\tau, \mu) = \phi(\nu, \mu)\, e^{-\tau/\nu}, \qquad (25)$$

are sought *a priori* with an important normalization condition, also set *a priori*:

$$\int_{-1}^{+1} \Phi(\nu, \mu)\, d\mu = 1 \; . \qquad (26)$$

Equation (18) becomes:

$$(\nu - \mu)\, \Phi(\nu - \mu) = \frac{\nu}{2} \sum_{\ell=0}^{N} \omega_\ell P_\ell(\mu) \int_{-1}^{+1} \Phi(\nu, \mu) P_\ell(\mu)\, d\mu \; . \qquad (27)$$

The two equations (21) and (25) are seen to be equivalent in form. The present method differs from the preceding one at this point. In place of approximation of the integral indicated above, a series of functions is defined:

$$g_\ell(\nu) = \int_{-1}^{+1} \phi(\nu, \mu) P_\ell(\mu) d\mu . \tag{28}$$

which obviously are calculated on the basis of a recurrence relation derived from that linking 3 consecutive Legendre polynomials:

$$\nu(2\ell + 1 - \omega_\ell) g_\ell(\nu) = (\ell + 1) g_{\ell+1}(\nu) + \ell g_{\ell-1}(\nu). \tag{29}$$

The first term, in accordance with equation (26), is:

$$g_0(\nu) = 1 . \tag{30}$$

The process is continued by defining the function:

$$g(\nu,\mu) = \sum_{\ell=0}^{N} \omega_\ell g_\ell(\nu) P_\ell(\mu) ; \tag{31}$$

equation (27) becomes

$$(\nu - \mu) \phi(\nu, \mu) = \frac{\nu}{2} g(\nu, \mu). \tag{32}$$

When ν falls within the range (-1, +1), $\nu - \mu$ is cancelled out. The expression for the solution sought in this range is then

$$\phi(\nu,\mu) = \frac{\nu}{2} P \frac{g(\nu, \mu)}{\nu - \mu} + \lambda(\nu) \delta^\circ(\nu - \mu) , \tag{33}$$ /1.3.7.

in which δ° denotes the Dirac distribution.

P indicates that the principal Cauchy value of the following expression is to be adopted in integration; $\lambda(\nu)$ is calculated on the basis of the normalization condition (equation 26):

$$\lambda(\nu) = 1 - \frac{\nu}{2} P \int_{-1}^{+1} \frac{g(\nu, \mu)}{\nu - \mu} d\mu. \tag{34}$$

There is thus a continuous set of singular eigenfunctions within the range (-1, 1). A finite number M of discrete eigenfunctions satisfying equation (26) is also found outside this range:

$$\phi(\pm \nu_i, \mu) = \frac{\nu_i}{2} \times \frac{g(\pm \nu_i, \mu)}{\nu_i \mp \mu} , \text{ with } L = 1, 2, \ldots M \text{ et } 2M \leqslant 2N + 1. \tag{35}$$

The values of ν_i are determined by the normalization condition (equation 26).

To recapitulate, functions $\phi(\nu, \mu)$ sought form a set consisting of a continuous subset and a subset of discrete elements.

The great value of this method lies in the fact that this set is complete and that the eigenfunctions are orthogonal [15, 18, 19, 50].

function L*, equations (39) and (40) then making it possible to find $L(\tau, \mu)$. Mathematically the problem is reduced to solution of an integral equation with respect to L*, which is easily obtained by replacing the expression for $L(\tau, \mu)$ of equations (39) and (40) in equation (38). This calculation principle is widely used in the case of spherical scattering, since the integral equation is simple, but involves substantial anisotropy. The mathematical developments are complex although possible. However, certain methods which have been described are indicated below without being discussed in detail.

- Simple Model of R. W. Preisendorfer

This model is described in [70]. The basic hypothesis consists in setting

$$L^*(\tau, \mu) = L^*(0, \mu)\, e^{-k/\tau_c} \qquad (41)$$

in which $L^*(0, \mu)$ is assumed to be known (initial conditions) and k is a constant independent of the depth which may be determined either by another method or by experiment. The expression for $L(\tau, \mu)$ is then easily found by means of equation (39), for example. This model is found to be very practical above all for study of the contrasts of submerged objects [22].

- Successive Approximation Method

This method may be understood directly on the basis of equations (38), (39), (40). A probable value is assigned *a priori* to luminance $L(\tau, \mu)$, for example by setting

$$L_o(\tau, \mu) = L_o(0, \mu)\, e^{-\tau/\mu} \qquad (42)$$

By use of an appropriate mathematical method determined by the form imparted to the scattering indicatrix, function $L_o^*(\tau, \mu)$ is calculated by means of equation (38), and then $L_1(\tau, \mu)$ is calculated by means of equations (39), (40).

The first approximation is thereby obtained and the process is repeated in order to arrive at an increasingly precise solution (convergence generally takes place). The difference between the nth approximation and the real solution depends, of course, on the method selected for representation of the scattering indicatrix.

- Spherical Harmonic Method

In this method solution $L(\tau, \mu)$ is sought in the form of development into a series of Legendre polynomials, the scattering indicatrix itself being developed in this form. The homogeneous transfer equation is then solved by use of the orthogonality properties and the recurrence relations between the Legendre polynomials. Different constants are involved; in order to calculate them use is made of the equations (38) and then (39) and (40), which take the boundary conditions into account.

This method is described in [40] for isotropic scattering and in [44] for anisotropic scattering. In [96] anisotropic scattering is also considered and an interesting comparison is made with the discrete ordinate method, these two methods ultimately yielding the same result.

- Other methods involving calculation of variations [40] or the properties of supplementary mathematical functions [16] may be used, but they are actually of interest only in cases in which the scattering is isotropic, and for this reason are not described here (see Section I.1.).

II.1.4. Approximate Methods for a Scattering Indicatrix Sharply Pointed Forward

These methods appear to be particularly well suited for the seawater medium, since in the approximations they make use of the fact that the scattering indicatrix is extremely sharply pointed forward. They are generally used when the source luminance is unidirectional over the entire surface (as in the case of sunlight, for example) and are valid only for small angles θ (generally speaking, $< 30°$).

Two types of methods are possible, methods employing the perturbation technique and those using Fourier transforms.

- Perturbation Methods

A detailed description of these methods is to be found in [96, 77, 79]. The general principle is that of finding the solution for $L(\tau, \mu)$ in the form

$$\boxed{L(\tau, \mu) \equiv L_o(\tau, \mu) + L_1(\tau, \mu) + \bar{L}(\tau, \mu)} \tag{43}$$

in which L_o is the light which has not been scattered and thus is known, L_1

corresponds to the first approximation, and $\bar{L}$ is the residue which may be estimated once L_1 has been calculated. It is assumed that optical depth τ is small, so that $L_1 \gg \bar{L}$. This makes it possible to write the transfer equation for L_1 with the appropriate boundary conditions only, and then for $\bar{L}$. The point of the indicatrix is introduced into the solution of these equations; for example, in equation (38) $L(\tau, \mu')$ is developed into a Taylor series around μ:

$$L(\tau, \mu') = L(\tau, \mu) + \frac{\partial L(\tau, \mu)}{\partial \mu}(\mu' - \mu) \ldots \qquad (44)$$

Once the scattering indicatrix has been developed in terms of Legendre polynomials, equation (37) may then be employed for the classical perturbation calculation [96]. /1.3.9

This calculation is of value only for slight optical depths. As a matter of fact, when simple scattering is involved the light is scattered chiefly in direction $\mu' = \mu$ owing to the point of the indicatrix. The greater the extent to which multiple scattering occurs (this being the case when τ increases) the more necessary is it to advance the development to higher orders so that L will remain small in comparison to L_1, an indispensable condition for application of this perturbation technique. These considerations ultimately restrict the application of this method.

- Fourier Transform Methods

In the solution of particular problems such as determination of the distribution of luminances at a certain distance from a point source situated in a diffusing and absorbing medium (point spread function), the approximation of small angles, which is possible owing to the sharply pointed indicatrix, makes it possible to put the integral occurring in the transfer equation into the form of a convolution product. In this case the solution is simplified in the Fourier space. This technique has been utilized, for example, by Wells [97] and by Dolin [20, 21].

II.2. "Natural Solution" Methods

The various methods based on evaluation of the successive orders of scattering have been grouped under this heading. The basic physical concept is obvious: photons scattered n times undergo simple scattering more than do photons scattered (n - 1) times arriving at the same point in the medium.

II.2.1. Principle

The principle underlying these methods is a very simple one.

- The luminance transmitted directly from the source is calculated at each point P of the medium and for each direction D: $L_o(P, D)$.

- Once $L_o(P, D)$ is known it is possible to calculate the luminance scattered in direction D from each point P'

$$L_1^* (P'D) = \int_{4\pi} L_o (P'D') \beta (P', D, D') d\omega' = R \; L_o(P'D') ; \qquad (45)$$

The scattering indicatrix is, of course, assumed to be known. R denotes the operator

$$\int_{4\pi} (\quad) \beta (P', D, D') d\omega' .$$

- The first order luminance at point P is derived from L_1^* by taking into account the attenuation between point P' at which simple scattering occurred and point P:

$$L_1 (P,D) = \int_0^{PP_\ell} L_1^* (P', D) T (|PP'|) d (|PP'|) = T \; L_1^* (P', D) ; \qquad (46)$$

$|PP'|$ represents the geometric distance between points P and P' and T is the attenuation undergone by a beam over distance PP'. If distance $|PP'|$ is r and if the optical characteristics are constant over path PP',

$$T (|PP'|) = e^{-cr} ;$$

P_l is the point situated at the boundary of the medium and in direction D (Figure 4).

Equation (46) may be written as follows:

$$L_1 (P,D) = T R L_o (P, D) = S L_o (P,D) \qquad (47)$$

- The higher order luminance is calculated in the same way.

- The real luminance sought is obviously the sum of the different orders:

$$L (P,D) = \sum_{i=o}^{\infty} L_i (P,D) \qquad (48)$$

A formal study of this natural solution, and in particular proof that the solution found is indeed the solution of the transfer equation, are to be found in the book by R. W. Preisendorfer [71]. A significant advantage of this method is represented by the fact that it provides directly the relative importance of the luminance orders. /1.3.10

II.2.2. Properties of Luminance Orders

Some of the simple properties are indicated in what follows.

- The concept of order is a relative one, since it defines the zero order as being the luminance coming directly from the source. If, for example, the source is represented by the sky, it is obvious that the light coming from this source has been scattered in the atmosphere. If the atmosphere were to be included in the medium, the breakdown into luminance orders would be different but the result would be the same.

- Operators R and T are linear relative to the luminance. Thus if surface calculation is performed for the same medium and for two types of surface distribution, the result corresponding to the total distribution is found by simple addition.

- When the scattering indicatrix does not change in form, operator R is linear relative to ratio b/c. As a matter of fact, equation (45) may be written in the form

$$L_i^{*}(P,D) = b/c \int_{4\pi} L_{i-1}(P,D')\, \beta_n(D, D')\, d\omega' ; \tag{49}$$

in this instance the coordinates of P are expressed in optical lengths and the medium is assumed to be homogeneous as regards the optical characteristics. On the basis of this property it is quite easy to demonstrate the validity of the relation [75]

$$L_i^{b/c}(P,D) = (b/c)^i\, L_i^1(P,D), \tag{50}$$

in which luminance $L_i^1(P, D)$ is the luminance of order i calculated on the assumption that the medium is not an absorbing one (b/c = 1). It is consequently sufficient to calculate the orders of luminance for the non-absorbing case. The total luminance for every value of b/c, if the medium is illuminated by the same source, is then found by means of equation (50). This property greatly enhances the value of employing the natural solution in the case of seawater, since the longest calculations may be performed merely once for various types of sources).

Another consequence of equation (50) is that the contribution made by luminance of order i to the total luminance is the greater, at the same optical depth, the more greatly absorbing is the medium (the smaller is b/c). Application of this to the case in which only zero order luminance is a carrier of information (unscattered light) reveals that this information will undergo the less destruction by scattering, for the same optical depth, the smaller is ratio b/c. (Ratio b/c may be varied, for example, by appropriate choice of the wavelength.)

II.2.3. Practical Application

- The natural solution method is applicable to all types of initial conditions. However, the total number N of luminance orders calculated is of necessity limited, either for lack of time or for lack of computer memory capacity. The approximate luminance obtained is

$$L_{ap} = \sum_{i=0}^{N} L_i = L - \sum_{i=N+1}^{\infty} L_i . \tag{51}$$

The higher orders of luminance are generally small in comparison to the first orders for slight optical depths. On the other hand, the greater the depth the more substantial are the orders higher than N and consequently the greater is the increase in the total error. This method thus yields exact results for slight optical depths, the limit of application obviously being determined by number N and by ratio b/c (see equation 50).

Another advantage of this method is the possibility of using the real diffusion index in the form of a numerical table, without the need for representing it by a mathematical development.

- This method has been applied to the case of seawater up to the first order [41, 38] and up to the third order [86] for the penetration of light deriving from the Sun only. It has also been applied to seawater up to the 21st order, with a realistic experimental indicatrix and for various types of initial distribution [75]. These last-named calculations make it possible to arrive at knowledge of the distribution of luminances down to 8 to 10 optical depths with good accuracy. It has been found in particular that for the penetration of daylight luminances of an order higher than 2 occur (depending more or less on the value of b/c) as soon as the optical depth exceeds unity. This emphasizes the fact that calculations in which only simple scattering is taken into account in a layer of a thickness greater than one optical half-depth remain highly approximate.

II.3. Methods Making Direct Use of Invariance Principles

Application of the invariance principles has made it possible to achieve exact solution of a large number of problems, particularly albedo problems for which the preceding methods yielded only approximate solutions. Preisendorfer considers them to be the basic concepts of any radiation transfer theory [72]. A detailed account of the methods is to be found in the works cited in the bibliography [16, Chapter VII] [71, Section 23]. /1.3.11

II.3.1. Invariance Principles

The following is the physical viewpoint. The medium is divided into layers, which are generally but not necessarily planes. Study is made not of the physical phenomena taking place inside a layer but only of the interactions between this layer as a whole and the radiation flux. Under these conditions the layer of the medium reflects diffusely on the side on which the luminance enters and also transmits diffusely on the opposite side. As a result of simple conservation of energy (Figure 5) two equations are obtained,

$$\left.\begin{aligned} L_+(a) &= L_+(b)\,T(b,a) + L_-(a)\,R(a,b) \ , \\ L_-(b) &= L_+(b)\,R(b,a) + L_-(a)\,T(a,b) \ . \end{aligned}\right\} \quad (52)$$

Luminances directed upward are identified by a plus sign and those directed downward by a minus sign.

The entering luminances are $L_-(a)$ on the upper surface of the layer and $L_+(b)$ on the lower surface. The other two types of luminances are emerging ones. The operators R(i, j) and T(i, j) are respectively the diffuse reflection and transmission operators, and the luminances to which they apply enter through surface i. These operators, which are of the integral operator type, depend on the optical characteristics inside the layer, that is, R and T are functions of c, β(P, D, D'), and on the thickness of the layer.

Group of equations (52) is an expression of the invariance principles. The fundemental invariance principle, which was first stated by Ambartsumyan

[1], is in reality expressed as follows: in a plane-parallel medium of infinite optical thickness the emerging luminance does not vary if any thickness is added to or subtracted from this medium. Chandrasekhar [16] subsequently extended reasoning of this type to the case of media of finite optical thickness and formulated four additional principles for them. Lastly, the various forms of these principles for all media are to be found in the work by Preisendorfer [71].

It is to be noted that equations (52), while they express conservation of energy, are equivalent to the transfer equation. Once the problem has been formulated in this manner, all that remains to be done to solve it is to find operators R and T. This is accomplished by establishing the functional relations connecting these operators. Methods of two principal types are used, one based on continuous formulation and the other on discrete formulation (addition of layers).

II.3.2. Continuous Formulation

It is applied chiefly to plane-parallel media illuminated in one direction. It is discussed in detail in [16, Chapter VII].

- A group of four integral equations satisfied by operators R and T is first formed on the basis of the mathematical expressions of the invariance principles (which is not presented here). The optical thickness, τ, of the medium in question and the scattering indicatrix enter directly and explicitly into these equations.

To solve these equations the indicatrix is again developed into Legendre polynomials and an eigenfunction, spherical harmonic, or successive approximation method is applied. Chandrasekhar [16] has shown that the solutions are broken down into products of functions some of which depend only on the angle of incidence on the layer and others only on the angle of emergence (reflection or transmission). There are two such functions, which are conventionally termed $X(\mu)$ and $Y(\mu)$ when the medium is of finite optical thickness. When the medium is infinite only a single function is involved, that termed $H(\mu)$. A great amount of study has obviously been devoted to these functions.

The value of this method lies in the fact that it provides an analytical solution to radiation transfer problems. However, while it may be applied to nonspherical scattering (Chandrasekhar, S. Pahor, Lenoble, 1963), it nevertheless appears to be not as convenient as the numerical methods now that computers have been developed.

II.3.3. Discrete Formulation

In contrast, this formulation lends itself readily to computer calculations. The following is the principle of resolution (all the calculation stages are discussed in detail in [71, Section 70]).

The medium is divided into layers of equal thickness, and only evenly distributed discrete directions are considered. Equations (52) are then matrix equations. Matrix operators R and T are assumed to be known for an elementary layer. They can be determined by assuming that only simple scattering occurs in the layer if the latter is of small optical thickness, or could be determined by any other method yielding a precise solution of the transfer equation for optical thicknesses which are not too great (the natural solution, for example).

According to equation group (52), the emerging, reflected, and transmitted luminances can be found if the luminances entering the layer are known. A layer of equal thickness is added under the preceding layer. The luminance /1.3.12 emerging from the latter downward is entering luminance as regards the added layer, while operators R and T remain the same (if the optical characteristics are homogeneous in the medium). Thus all the luminances are calculated by iteration, by successive addition of layers of equal thickness.

Diffuse reflection and transmission operators for 2 layers of equal thickness are derived from operators R and T which are known for one layer. There are recurrence relations making it possible to obtain the operators corresponding to n successive layers. These relations are the equivalent of the functional relations established in the continuous formulation.

Hence it is possible, for a medium with given optical characteristics, to calculate the elements of the operator matrices corresponding at an equal optical depth to a whole number of elementary layers. To arrive at knowledge of the distribution of the luminances at the various optical depths it suffices to apply the matrix operators to the luminances imposed by the boundary conditions.

In this method the accuracy of distribution of the luminances depends directly on the dimensions selected for the matrices (and thus on the capacity of the computer employed). It further depends on the thickness selected for the elementary layer, since the error committed by assuming that the multiple scattering in it is negligible is augmented when successive layers are added.

It must be observed that the results obtained by R. W. Preisendorfer for 26 directions in space and for an elementary layer thickness of one optical half-depth are in close agreement with experimental data [71, Chapter 11).

It may be said in recapitulation that this method appears to be well suited to penetration of daylight into the sea, to the extent that no attempt is made to determine the continuous angular distribution of the luminances with precision. In particular, it makes it very easy to deal with cases in which the sky is not uniform but cloudy. This method is not limited to the case of daylight penetration, but in other problems (point source, laser ray) the number of matrices necessary for calculation increases considerably, the simplification made by the plane-parallel symmetry now disappearing.

II.4. Monte Carlo Methods

These methods are essentially numerical ones and the frequency of their use has increased apace with the development of computer capacity. The principle is that of following the progress of a photon penetrating a scattering-absorbing medium in order to learn its position and its direction at the place in the medium in which the detector is assumed to be located. Thus a large number of photons are followed and the final distribution sought is obtained by statistical means.

In these methods all the phenomena of interaction between photons and the medium must be expressed in the form of the law of probability, refraction and reflection at the air-sea interface, absorption, scattering, scattering angle, etc. As a matter of fact, it is necessary to know only the laws governing the elementary phenomena, as is the case with seawater.

As regards a photon, absorption behaves as an all-or-nothing phenomenon and the probability that a photon will disappear is a function of the path that it has traveled in the medium. It is thus necessary to record this path. This is often simulated by assigning a starting statistical weight which decreases as a function of the path traveled with each event undergone by the photon. The photon is considered to have disappeared when its weight drops below a preassigned value.

Two phenomena must be determined for scattering: the place at which the photon is scattered and the angle indicating the departure of the photon from its initial path. It is similarly necessary for reflection and refraction to determine by a probability law if the photon is reflected or refracted.

To apply these probability laws it is necessary to have a random number generator which is resorted to on each occasion that an event is undergone by the photon. With this random number and knowledge of the probability law assigned to the event, complete determination is made of the progress of the photon (scattering angle, reflection, etc.).

In these methods the scattering indicatrix is generally tabulated and may thus be very markedly pointed forward.

There is in reality a wide variety of methods, in some of which the random process is applied to all the physical phenomena and in others only to some of the phenomena, with an analytical process applied to the others.

These methods are very general ones, and models of complex scattering media may be constructed. A very complete ocean-atmosphere model with clouds for various altitudes of the Sun is presented in references [57-65]. One disadvantage of these methods is due to the fact that they are purely numerical ones and that the physical laws are consequently more difficult to reveal. Thus it is necessary to repeat the calculations in their entirety in order to determine the influence of each parameter. Another disadvantage is that refinement of the calculations is a lengthy process requiring very careful verification, since the accuracy of calculations of this type can be known only when they have been completed [58]. In addition, this accuracy depends directly on the time devoted to the calculations, since it is determined by the number of recorded "cases" of photons penetrating the medium under study.

II.5. Theories Taking Fluctuations Inside the Medium into Account

It was assumed in the foregoing that the optical characteristics of the medium were fixed and stationary (refraction index, scattering indicatrix, absorption, attenuation, etc.). Under natural conditions, however, the refraction index of the atmosphere or of the ocean is not homogeneous and exhibits fluctuations in time of varying scale. A medium is said to be random or turbulent. It could be assumed that these fluctuations are stationary in nature, and a scattering indicatrix could be derived and the preceding methods applied. In reality these methods are not suited to this /1.3.13 case, for which several theories have been elaborated. The practical problem consists in learning the response of a detector having a certain time constant and a certain field angle to the intensity of a source (generally a remote one). A turbulent medium is situated between the source and the receiver. If this medium does not fill all the space between the source and the receiver it is easy to arrive at the first case by simple optical geometry calculations.

These theories have generally been applied to the propagation of a laser beam or to explain scintillation through the atmosphere, which is assumed to be fluctuating but transparent. In other words, there are no particles in the physical sense of the word, the scattering then being due only to index heterogeneities. In reality there is no physical discontinuity between the scattering by particles and that generated by index heterogeneities. Logically both phenomena should be taken into account, above all in the case of seawater (see, for example, [102, 76, 30]). Only the influence of index heterogeneities is considered in what follows.

It is not possible in this survey to deal exahustively with the problems raised by the random scattering processes, to which much study has been and is being devoted, especially in atmospheric optics. The works on which the majority of the current studies appear to be based are those by V. I. Tatarskiy [87, 88] and Born and Wolf [11]; many articles have been published in recent years [6, 7, 8, 13, 14, 23, 25, 29, 32, 33]. Up to the present, however, there have been only few applications to the case of seawater [102]. Discussion here will be confined to brief indications of the manner in which the problem is formulated and in what terms it has been solved.

The basic equation [87, p. 60] which describes the propagation of an electromagnetic wave the wavelength of which is very small in comparison to the dimensions of the turbulent cells, has been set up on the basis of a Maxwellian equation.

$$\nabla^2 \vec{E} + k^2 n^2 \vec{E} = 0 , \tag{53}$$

in which n is the index and $\vec{E}$ is the electric field vector representative of the wave.

In order to determine the illumination at a point situated at a certain distance from a source it is enough to solve the scalar equations of the components of the electric field. One component being u, the solutions are sought in the form

$$u = Ae^{is} ,$$

in which A is the amplitude and s the corresponding phase.

Since the refraction index undergoes random variation, the amplitude and the phase of the wave will also be characterized by random variations. To describe the distribution of illumination around a point use is made of the mutual coherence function Γ [11, 55], which is a mean function in time:

$$\Gamma (x_1 , x_2 , \tau) = u(x_1 , t + \tau) . u^* (x_2 , t) ,$$

or analogous statistical functions, correlation functions, or structural functions [87]. (In the above equation x_1 and x_2 are two points situated in a plane perpendicular to the propagation of the wave.)

It is to be noted that function $\Gamma(x_1, x_2, o)$ represents the correlation between signals obtained at two points at the same time, and that $\Gamma(x, x, \tau)$ is the correlation between the signals at the same point at two moments separated by time τ. This function is thus related directly to the concept of coherence.

Methods of two types are discussed by various authors [32].

- Solutions to [53] are found in terms of A and s, and the correlation functions are then calculated for the phase and for the amplitude depending on the correlation functions for the index [87, 88, 103, 13].

- Some authors [8, 9, 32, 100] have established a differential equation based on the distance to the source and relating directly to the Fourier coefficients of function Γ; the problem is then to solve this equation.

Certain assumptions must be made in all these methods, ones such as slight variations in index, fluctuations in indices expressed in manageable mathematical forms, etc. In addition, the solution is valid only for a range of distances which generally are not too large. It is to be remembered that the results differ according to the values of simple criteria associated with the wavelength, the aperture of the initial beam, and the observation distance [13, 87, 46]. The coherence function is found to be asymptotic for large distances [9, 101].

II.6. Comments on Polarization

Throughout the foregoing light has been assumed to be nonpolarized. In reality, the majority of the methods described are equally applicable to study of light involving use of the four Stokes parameters [16, 71, 5, 4]. According to certain authors [28] it appears that the fact that the polarization of daylight is unknown does not greatly modify the results from the viewpoint of illumination. It appears that no account need be taken of it when the phenomenon studied is specifically modification of the state of polarization. A survey of the polarization of daylight has recently been made by A. Ivanoff [35].

III. Applications of the Methods /1.3.14

In this section a description is given of certain problems which are of interest in oceanography and the application to which of the preceding methods permits theoretical forecasting of the results. The list of not an exhaustive one, but the few questions dealt with demonstrate the supplementary information which may be derived from application of the theories to the experimental data.

III.1. Pentration of Daylight into the Sea

Knowledge of this penetration with the greatest possible accuracy is highly important in biological oceanography in all matters concerning the primary production of oceans. The ideal would be to measure the luminances, but although such measurements are not impossible [84, 95], their generalization appears to be difficult and probably not necessary. Knowledge of the extinction coefficients of plane or spherical illumination is very often enough for practical applications. However, these coefficients are not optical characteristics, since they depend additionally on the surface illumination conditions. Thus for the purpose of efficient forecasting of daylight penetration it is necessary to undertake to establish relations between the extinction coefficient and the optical characteristics (Section I.1.).

The definition itself of extinction coefficient K is

$$K = \frac{1}{E}\frac{d\mathrm{Log}E}{dz} \tag{54}$$

in which E represents the plane or the spherical illumination. These coefficients obviously have values intermediate between the absorption coefficient and the attenuation coefficient.

To find the relations cited above one may calculate the distribution of the luminances for different types of surface illumination conditions and for different optical characteristics; illumination E and thus K are derived from them by integration. For the sake of greater simplicity it is generally assumed that the medium is plane-parallel (see I.1) and homogeneous.

Many calculations employing various methods have been proposed for distribution of luminances in the sea: the discrete ordinate method in the third approximation [42, 43], the eigenfunction method [24], that utilizing the great asymmetry of the scattering indicatrix [78, 80], another applying the principles of invariance [71], the Monte Carlo method [64, 65], and the natural solutions [75]. However, these calculations are not systematic in nature, often presenting no more than a few particular cases. It is difficult to derive from them the influence of external conditions (altitude of the Sun, overcast sky) and of depth on the luminances and consequently on the extinction coefficients.

As regards the influence of the altitude of the Sun on these last-named values, it appears not to be substantial [10, 39] at least for altitudes which are not too low. As for the variation of K with depth (in a homogeneous sea), scarcely any theoretical study has been devoted to it except apparently in references [66, 67, 69], in which a relation is established between the extinction coefficient and the ratio of illumination of a plane to spherical illumination, which itself undergoes but little variation. A simple and interesting theory is proposed in [99]. It is based on the invariance principles and provides a relation, dependent on depth, between K and the ratio of forward scattering to back scattering. It does not appear, however, to be directly applicable with precision to the case of seawater, in which the vertical distribution of the particles differs from those considered.

It is true that a large number of *in situ* measurements of the extinction coefficient are available, but only few of the measurements were performed simultaneously with measurement of the optical characteristics of the medium. In addition, the latter generally varied with the depth, and thus it would be difficult to determine even by way of experiment the influence of depth on the relations sought. Lastly, this is a point to which systematic application of the theory should prove to be useful.

Particular Case of Asymptotic Conditions

Such relations are currently known when the optical depth is sufficiently great. It is shown [68, 31] that the distribution of luminances tends toward an asymptotic form of revolution about the vertical. Under these conditions the extinction coefficient depends only on the scattering indicatrix and on b/c. Asymptotic conditions exist whatever the scattering indicatrix and the surface conditions if ratio $b/c < 1$ (passive medium), and nearly all the methods permit study of them. They have been revealed by way of experiment chiefly in the laboratory, in artificial media representative of seawater [92, 94]. The mathematical methods, for example the discrete ordinate method, very readily yield the relations between the extinction coefficient and the optical characteristics, and a very complete study is to be found in references [27, 28, 106]. A simpler method has been employed [74, 73] in the case of seawater by use of the real indicatrix (I.1). The close agreement between these last-named results and the results of experiments [90] makes it possible to obtain dependable results when they are used for the case of seawater.

In short, the relationships between the extinction coefficients and the optical characteristics are known only for the case of asymptotic conditions. For slight depths such relations can be obtained only by the discrete ordinate method or by the natural solution, which is well suited to the case of seawater, in which the scattering indicatrix due to a particle is of a form which may be considered to be approximately constant.

III.2. Aerial Detection of Daylight Deriving from the Sea

Substances in suspension and in solution act to fix the color of the sea. Thus an attempt has been made to study this color from a distance, from air- /1.3.15 craft or satellites, in order to obtain a general view of the surface content of the oceans. This problem involves radiation transfer in the atmosphere, in the sea, and at the interface. A recent publication [3] deals with the question in a very specific and detailed manner.

The problem amounts to evaluation of the apparent luminance, L_z, of the sea at a certain altitude, z, for various wavelengths.

$$L_z = L_a^* + (L_r + L_u)\, T_a \tag{55}$$

L_a^* is the luminance scattered by the atmosphere in the direction in question from the source made up of the Sun and the sky.

L_r is the luminance reflected by the surface of the sea,

L_u is the luminance diffusely reflected by the entirety of the scattering and absorbing medium represented by seawater,

T_a is the transmission in the atmosphere.

The information sought is contained in L_u, and the other terms appear as spurious luminances.

Calculation of luminance L_a^* is a classical problem of radiative transfer in the atmosphere, while calculation of L_r involves the reflection of light rays on a surface which generally is not a plane one but fluctuates in time as a result of the effect of winds. A study of this subject is to be found in references [3], [38].

More detailed discussion is devoted here to the methods of theoretical evaluation of luminance L_u deriving from the sea. Among the various methods described in Part II, that making use of the invariance principles (II.3) would appear to be well suited to the problem. As a matter of fact, no attempt is made to learn the distribution of luminances in the medium, but only the overall effect of the medium on the incident daylight. Knowledge of the diffuse reflection operator is thus enough to provide an answer to the questions raised. However, the influence of the turbidity of the water and the wavelength, which are manifested in variation in ratio b/c, would be difficult to evaluate, in view of the fact that linear variation in this ratio does not appear in the solutions. A method such as this does not appear to have been used in the case of seawater.

On the other hand, the natural solution method directly reveals several interesting points [75].

- The contribution to the total backscattered luminance made by each order of luminance is the same when the medium is a purely scattering one. The relative contribution of luminance of order i to the total luminance is consequently proportional to ratio $(b/c)^1$ (see II.2.2.).

- Since the directly transmitted luminance becomes small in comparison to the diffuse luminance as soon as the optical depth exceeds several units, the luminance backscattered from the sea in the main reflects only the absorbing and scattering content of the surface layers (1 or 2 optical depths).

- Lastly, the spectral dependence of this luminance is associated directly with that of ratio b/c in the surface layers, since the larger is ratio b/c, the greater is the backscattered luminance for the same incident luminance value (equation 57).

These properties may be summarized by the following expression, in keeping with the notation presented in Section II.2.

$$L(0,\theta) = \sum_{i=0}^{\infty} (b/c)^i \, L_i^1(0,\theta) \qquad \pi/2 < \theta < \pi$$

$$L_i^1(0,\theta) \approx L_j^1(0,\theta) \approx f(\theta) \qquad \text{whatever the value of } j \tag{56}$$

$$L(0,\theta) \sim f(\theta) \sum_{i=1}^{\infty} (b/c)^i = f(\theta)\,\frac{b/c}{1-b/c} \quad ; \ \pi/2 < \theta < \pi \tag{57}$$

in which $f(\theta)$ is a function which depends on the nature of the surface illumination. It is to be noted further that the term $L_o^1(0,\theta)$ is taken as equal to zero, since there is no direct luminance pointed upward, the bottom of the ocean being assumed to be remote enough so that reflection effects may be disregarded.

Lastly, a very complete ocean-atmosphere model can be obtained by means of the Monte Carlo method, and thus numerical study may be made of the terms of expression (55) [65, 64]. Practical study has also been made of the influence of the bottom on ascending illumination, except in the last-named reference. As soon as the optical length exceeds three, this method is no longer sensitive, even when the albedo of the bottom is taken as equaling unity.

III.3. Artificial Sources /1.3.16

This section concerns theoretical study of the obligation of electromagnetic waves emitted by artificial sources such as lamps or a laser. This problem is more or less directly related to that of transmission of information over optical channels and is of considerable interest for this reason. This subject is dealt with in other works; hence discussion here will be confined to several simple sources which have already been studied. A certain number of studies [48, 49, 55] provide the basic concepts relating to the optical values to be determined in order to ensure the transmission of an image. These values may be derived from the response of the medium when it is illuminated either by an omnidirectional point source or by an unidirectional point source (the laser ray is a good approximation). It is to be pointed out that examination of these particular source cases is of actual value only to the extent that the medium is considered to be linear. In addition, the validity of such an assumption has been discussed in [30] for the case of seawater.

III.3.1. Omnidirectional Point Source (See Figure 6)

The problem of interest is to learn the distribution of luminances at a certain distance R from the source or its Fourier transform of angular frequencies in space, which, in the case of incoherent light, is the modulation transfer function (MTF) [98].

Several methods have been used to solve this problem. They all involve approximations based on the fact that the scattering takes place for the most part at small angles. Mention may be made of the method of Wells, who provides an analytical solution and gives the MTF as a function of distance [97]. The following is the principle of calculation: in an elementary layer of the medium the distribution of the luminances at the exit from the layer may be derived from that at the entrance by a product of convolution with the scattering indicatrix (when the scattering angle is not large). The solution presents no difficulty in the Fourier space. Other methods have been proposed, ones which make use of the principles of invariance [104] or of Monte Carlo calculations [17]. They provide numerical results but not an analytical solution. A comparison of these methods has recently been published [105], and the uncertainty regarding the scattering indicatrix at very small angles [53] scarcely permits evaluation of the validity of the various approximations made. Lastly, the natural solution method might also be applied to this case and would yield directly the magnitude of the various scattering orders without resort being made to the small angle approximations.

III.3.2. Unidirectional Point Source (See Figure 7)

Since lasers have made their appearance a large number of studies have been undertaken on propagation of the light generated by such a source. Mention has already been made (II-5) of several theories on this subject when the scattering is due only to fluctuation of the refraction index. The effect of scattering of this type is appreciable for high angular frequencies. The effect due to the scattering indicatrix of seawater particles, which is not necessarily random in nature, has also been studied from the theoretical viewpoint and *a priori* concerns lower angular frequencies. The problem is to determine the distribution of illumination in a plane perpendicular to the initial path of the light beam.

In accordance with the principle of reciprocity, the problem is the same as that dealt with in Section III.2.1. [26]), but the formulation and the solution are different. The Monte Carlo method has been used and the numerical results, compared with the experimental data, are presented in such a way that they can very easily be exploited [26]. (In particular, it is possible to infer from them the illumination deriving from a cone of light, and the extinction coefficient valid in this case is compared with the optical characteristics.)

The transfer equation corresponding to the problem formulated has been solved by again making certain plausible approximations (small angles, use of Fourier transforms) [3, 12]. The theoretical results are to be compared with those obtained by way of experiment; in particular, two empirical formulas are proposed, one for illumination on the axis of a source characterized by a certain amount of divergence [22], and the other for illumination in a plane perpendicular to a unidirectional source [37]. It has also been found by way of experiment that illumination in the vicinity of the point of impact of a beam is Gaussian in distribution [47]. Lastly, certain calculations have been proposed which are based on the fact that scattering at small angles may be considered to be a random phenomenon [102, 5a, 23].

III.3.3. Extended Sources

There is no single method which may be applied to gain precise knowledge of the propagation of light coming from a nonpoint source (this being necessary in photography, for example). The study is of necessity subdivided into consideration of several regions of space frequencies in space: that of the high frequencies (observation of minute details), which, as we saw in II.5, involves large-scale index fluctuations and the scattering indicatrix of particles at very small angles (less than a few hundredths of a radian); that in which the scattering indicatrix of the particles concerns medium angles (III.3.1.); and lastly, the region of low space frequencies in which scattering as a whole takes place. Study of the last-named region concerns examination of the general contrasts of an object of considerable vertical extent. The simple model of Preisendorfer (II.1.3.) may then yield acceptable results, while the approximations employed in the methods cited in III.3.2. and III.3.3. become less valid.

III.4. Inverse Problems

In the problems considered in the foregoing, which are described in general as "direct" ones, the optical characteristics (scattering and attenuation coefficients, scattering indicatrix) were assumed to be known, as were also the initial conditions. The distribution of the luminances or of the quantities derived from them were determined on the basis of these data. In the "inverse" problems the optical characteristics of the medium are inferred from the initial conditions and from the distribution of the luminances within the medium. The interest of the latter methods is apparent when the optical characteristics of the medium are unknown or can be measured only with difficulty, and when on /1.3.17

the other hand the experimental results provide direct or indirect access to the distribution of the luminances.

There are two methods of dealing with these problems. The relationships between optical characteristics and luminance distribution are sought by analytical means, this necessarily leading in the case of seawater to an attempt to arrive at the scattering indicatrix in the form of an expansion into Legendre polynomials or in a form approximating a simple function. One may also select an indicatrix *a priori*, perform the calculations by the previously described methods, compare the results obtained with the experimental results, subsequently modify the indicatrix, and begin again in order to obtain a satisfactory result. This is a process of iteration.

In all instances it is necessary to define the form of the indicatrix *a priori*, and it ultimately is never certain that it is the real form, even when the results are good. By way of example several typical inverse problems are discussed in what follows, two of which require no knowledge of the scattering indicatrix and thus may be fully solved.

III.4.1. Absorption Coefficient

This is a simple case making direct use of equation (11b). When the medium is a plane-parallel one, this being a good approximation as regards the penetration of daylight, this equation is written as follows:

$$\frac{dE_d}{dz} - \frac{dE_u}{dz} = aE_o , \qquad (58)$$

in which E_d and E_u represent respectively plane descending and ascending illumination. These illuminations are measured on a routine basis, and the scalar illumination may also be measured in this manner, for example, by use of a spherical scattering collector. The absorption is thus derived from these measurements. This absorption coefficient may also be obtained as the difference between the attenuation coefficient and the total scattering coefficient, but the result is much less exact, since the absolute value of these coefficients is very difficult to determine.

III.4.2. Attenuation Coefficient

By application of transfer equation (9) to horizontal luminance in a plane--parallel medium, we obtain

$$c = \frac{L^*}{L} . \tag{59}$$

This is the black screen method [41, 34]. The attenuation coefficient may be measured by comparing the horizontal luminance of water at a depth with the apparent luminance of a black screen situated a known distance from the observation point. This method has also been employed in the atmosphere [22b].

III.4.3. Form of Scattering Indicatrix

This indicatrix may be derived from the distribution of the luminances, and in particular from that present under asymptotic conditions. As a matter of fact, each form of indicatrix has a boundary distribution of its own corresponding to it. The two types of approach indicated above may be applied, but we will consider here an analytical method described at length in reference [106]. The scattering indicatrix is sought in the form of an expansion into Legendre polynomials. This expansion is first applied to the known distributions of the luminances, and the different moments of the angular distribution of the luminances are then calculated. (The zero order moment is spherical illumination, and the first order moment is plane illumination.) It is demonstrated that recurrent relationships exist between the different moments and the coefficients of expansion of the indicatrix. The latter values may thus be calculated step--by-step.

The principle of this method has actually been applied [81, 28], but it must be observed that, since the scattering indicatrix is highly pointed, the expansion must be carried to fairly high order. This requires very precise knowledge of the luminances. It is to be noted that, if the asymptotic conditions are used, only the first terms of the expansion can be obtained with precision. As a matter of fact, variations in the indicatrix exert but little influence on the form of luminance distribution [74, 107]. It is shown in [107] that the first 5 to 10 terms suffice for determination of the asymptotic distribution with precision. This means, again, that this method can be employed to determine the form of the indicatrix only for angles which are not too small.

In reality a good direct knowledge of the indicatrix is available in this region, and thus these calculations are generally not necessary.

III.4.4 Scattering Indicatrix at Small Angles

The situation is entirely different as regards small angles, for which only very few measurements are available. In addition, it is in this region ($< 1°$) that the form of the indicatrix may vary [53] according to the scale considered. Physical measurements permitting determination of the form of this indicatrix by an inverse method should be sensitive to change in the luminance distribution at small angles. Hence study must be made, for example, of the propagation of a collimated beam or of the modulation transfer function [17]. It has unfortunately been found that there appear in these problems two physical phenomena the effects of which are superimposed: optical scattering (particles, molecular scattering) and index fluctuations. Thus the inverse problem is complicated by the fact that it is not easy to distinguish the two types of scattering on the basis of experimental data (distributions of illumination perpendicular to /1.3.18
the beam). Other information on the patterns of index fluctuation must be obtained independently. Recent developments in study of seawater transfer should permit exploration of this region.

Findings

A broad spectrum of theoretical methods are available for calculation of the propagation of electromagnetic waves in seawater. The following conclusions may be drawn from the foregoing discussion:

Only a few papers have translated these theories into calculations, and above all into calculations which may be used directly for forecasting. While the analytical methods have been developed somewhat of late by the use of eigenfunctions, the peculiar nature of these functions often hampers their application. The numerical methods, in turn, which are well suited to computer calculations, are found to be highly practical, but an effort must be exerted to present the results in a form which can be utilized by persons who do not have direct access to the programs, which always take a long time to elaborate. One goal which could be assigned to the calculations would be, for example, the task of relating the extinction coefficients to the optical characteristics by making objective evaluation of the influence of the initial conditions and their importance.

The form of the scattering indicatrix at very small angles continues to be a problem which must be solved if it is desired to forecast the limits of information transmission in seawater, and it would appear that it is at the present still necessary to compare the theoretical with the experimental results in order to determine the influence of index fluctuations.

Considering the relatively constant form of the scattering indicatrix, we have also become aware of the importance assumed by the b/c ratio in explanation of penetration by electromagnetic waves. Thus in order to make a comparison among different measurements it is useful to determine this ratio characteristic of experimental conditions.

Note: It must be pointed out that the papers by G. V. Rozenberg have not been analyzed in this survey, for documentation reasons. In addition, the publication of an important treatise on Oceanographic optics by R. W. Preisendorfer is announced.

FIGURES

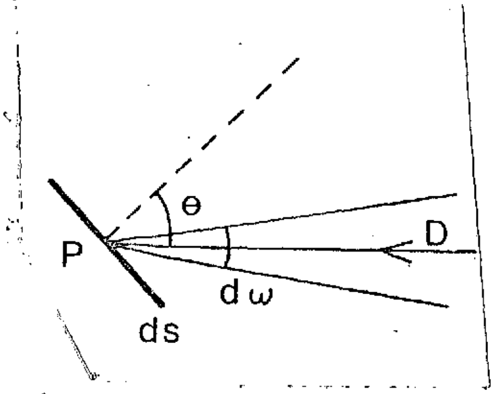

Figure 1.

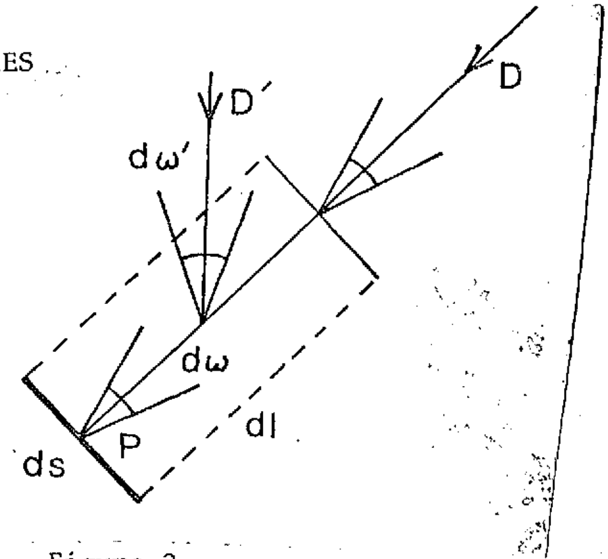

Figure 2.

/1.3.24

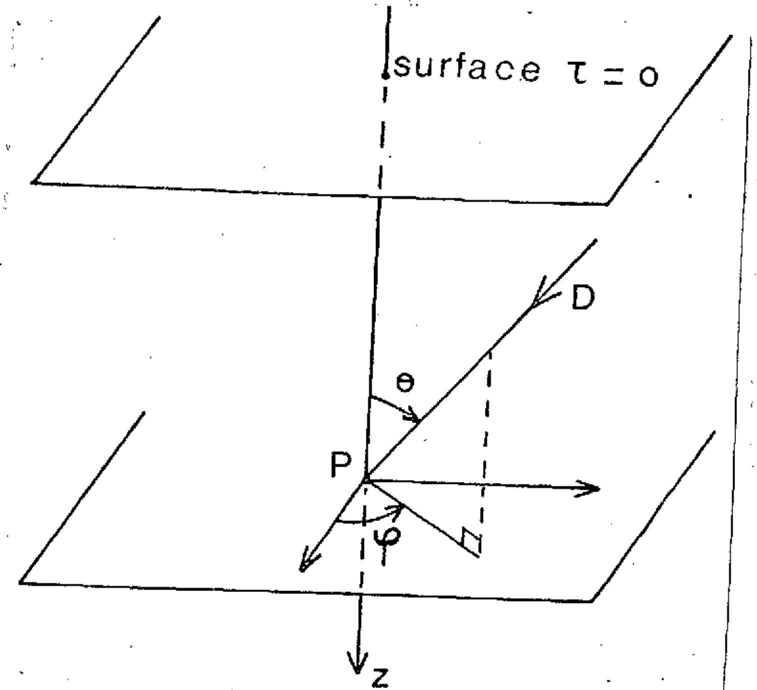

Figure 3.

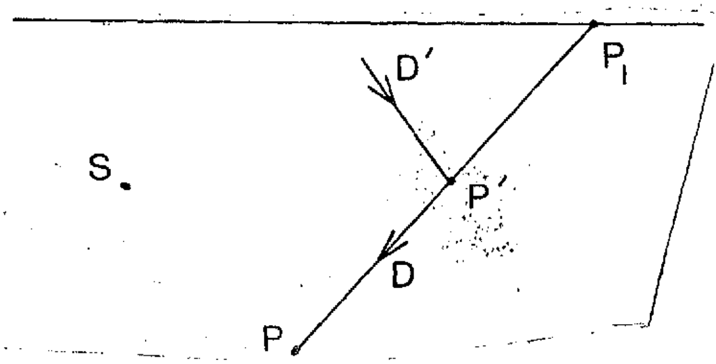

Figure 4. Source S is Here a Point Source but May be of any Nature; for Example, it Could be Formed by Surface Luminance Distribution.

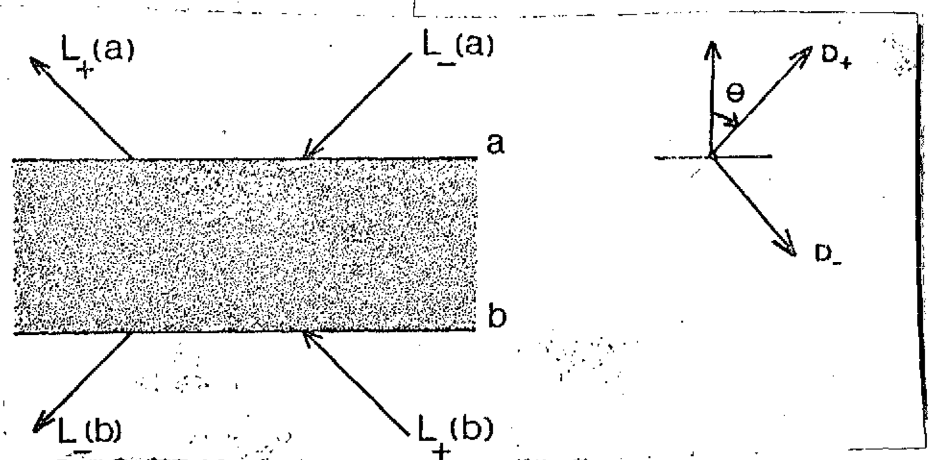

Figure 5.

/1.3.25

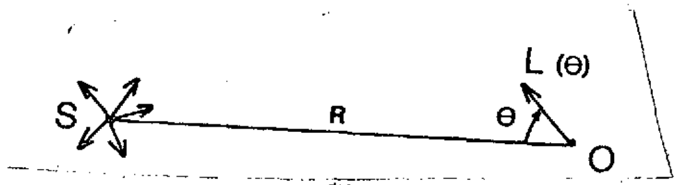

Figure 6. The Isolume Point Source is at S; Observation is Conducted at O.

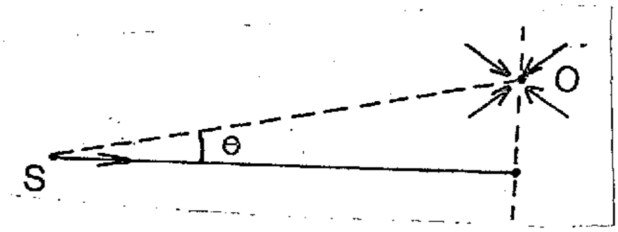

Figure 7. Unidirectional Point Source.

REFERENCES

1. Ambartsumyan, V. A., "On the Problem of Diffuse Reflection of Light in Planetary Atmosphere," *Smithsonian Inst. Ast. Papers*, Vol. 7, pp. 25-33, 1943. /1.3.19
2. Arnush, D., "Underwater Light Beam Propagation," *J.O.S.A.*, Vol. 62, pp. 1109-1111, 1972.
3. Austin, R. W., "The Remote Sensing of Spectral Radiance from Below the Ocean Surface," *Symposium on Optical Aspects of Oceanography*, Academic Press, Copenhagen, in press.
4. Beardsley, G. F., "The Polarization of Submarine Daylight at Near-Asymptotic Depths," *Journ. of Geophysical Research*, Vol. 73, pp. 6449-6457, 1968.
5. Beardsley, G. F., "Mueller Scattering Matrix of Seawater," *J.O.S.A.*, Vol. 58, No. 1, p. 52, 1968.

5b. Beck, C., "Transmission of Light through a Randomly Scattering Medium," Naval Air Development Center, Report No. NADC 72099, AE/, 1972.

6. Beran, M. J., "Propagation of Mutual Coherence Function through Random Media," *J.O.S.A.*, Vol. 56, No. 11, pp. 1475-1480, 1966.
7. Beran, M. J., "Propagation of the Coherence Function through Random Media," *J.O.S.A.*, Vol. 58, No. 3, pp. 431-432, 1968.
8. Beran, M. J., "Propagation of a Finite Beam in a Random Medium," *J.O.S.A.*, Vol. 60, No. 5, pp. 518-521, 1970.
9. Beran, M. J. and A. M. Whitman, "Asymptotic Theory for Beam Propagation in a Random Medium," *J.O.S.A.*, Vol. 61, No. 8, pp. 1044-1050, 1971.
10. Bethoux, J. P. and A. Ivanoff, "The Influence of the Altitude of the Sun on the Value of the Extinction Coefficient of Seawater," *C. R. Acad. Sci.*, Vol. 270, B, pp. 1079-1710, 1970.
11. Born, M. and E. Wolf, *Principles of Optics*, Pergamon Press, 1959; republished in 1965 and 1970.
12. Bravo-Zhivotosky, O. N., L. S. Dolin, A. G. Louchinin and W. A. Savelev, "On the Structure of a Narrow Light Beam in the Seawater," *Izv. Atm. and Ocean Phys.*, No. 2, pp. 160-167, 1962.
13. Carlson, F. P., "Application of Optical Scintillation Measurements to Turbulence Diagnostics," *J.O.S.A.*, Vol. 59, p. 1343, 1969.
14. Carlson, F. P. and A. Ishimaru, "Propagation of Spherical Waves in Locally Homogeneous Random Media," *J.O.S.A.*, Vol. 59, No. 3, pp. 319-327, 1969.
15. Case, K. M., "Elementary Solutions of the Transport Equation and their Applications," *Ann. Phys.*, Vol. 9, p. 1.23.
16. Chandrasekhar, S., *Radiative Transfer*, Dover Publ. Inc., New York, 393 pages; republished in 1950, Oxford University Press.
17. Chilton, F., A. D. Jones and W. K. Talley, "Imaging Properties of Light Scattered by the Sea," *J.O.S.A.*, Vol. 59, p. 891, 1969.
18. McCormick, I. Kuscer, "Half Space Neutron Transport with Linearly Anisotropic Scattering," *J. Math. Phys.*, No. 6, p. 1939, 1956.
19. McCormick, N. M. and I. Kuscer, "Biorthogonality Relation for Solving Half Space Transport Problems," *J. Math Phy.* No. 7, p. 2036, 1966.
20. Dolin, L. S., "Scattering of a Light Beam in a Layer of a Turbid Medium," *Bull (Izv) Higher Educ. Inst Radiophysics*, Vol. 7, No. 2, 1964.
21. Dolin, L. S., "Propagation of a Narrow Light Beam in a Medium with Strongly Anisotropic Scattering," *Bull (Izv) Vyssh. Uchebn. Zaved. Radiofizika (Higher Educ. Inst. Radiophysics)*, Vol. 9, No. 1, 1966.

22. Duntley, S. Q., "Light in the Sea," *J.O.S.A.*, Vol. 53, p. 214, 1963.
22b. Duntley, S. Q., A. R. Boileau and R. W. Preisendorfer, *J.O.S.A.*, Vol. 47, p. 419, 1957.
23. Duntley, S. R., W. H. Vulver, F. Richey and R. W. Preisendorfer, "Reduction of Contrast by Atmospheric Boil," *J.O.S.A.*, Vol. 53, No. 3, p. 353, 1963.
24. Feistein, D. L., K. R. Piech and A. Leonard, "Transport Analysis of Optical Propagation in the Undersea Environment," Agard Conference Proceedings, No. 77 on *Electromagnetics of the Sea*, No. 38.
25. Fried, D. L., "Diffusion Analysis for the Propagation of Mutual Coherence," *J.O.S.A.*, Vol. 58, No. 7, pp. 961-969, 1968.
26. Funk, C. J., "Multiple Scattering Calculations of Light Propagation in Ocean Water," *Applied Optics*, Vol. 12, No. 2, pp. 301-313, 1973.
27. Herman, M. and J. Lenoble, "Study of Asymptotic Conditions in a Scattering and Absorbing Medium," *Revue d'Optique*, Vol. 43, No. 4, pp. 555-557, 1964.
28. Herman, M. abd J. Lenoble, "Asymptotic Radiation in a Scattering and Absorbing Medium," *J. Quant Spectrosc. Radiat. Transfer.*, Vol. 8, pp. 355-367, Pergamon Press, 1968.
29. Ho, T. L., "Coherence Degradation of Gaussian Beams in a Turbulent Atmosphere," *J.O.S.A.*, Vol. 60, No. 5, pp. 667-673, 1970.
30. Hodgson, R. T. and D. R. Caldwell, "Application of Fourier Technics to Underwater Image Transmission," *J.O.S.A.*, Vol. 62, No. 12, pp. 1434-1438, 1972.
31. Hojerslev, N. V., "A Theoretical Proof of the Existence of a Constant Vertical Radiance Attenuation Coefficient in a Horizontally Stratified Ocean," *Institut for Fysik Oceanografi Report*, No. 2, 1972.
32. Hufnagel, R. E. and N. R. Stanley, "Modulation Transfer Function Associated with Image Transmission through Turbulent Media," *J.O.S.A.*, Vol. 54, p. 52, 1964.
33. Ishimaru, A., "Fluctuations of a Beam Wave Propagating through a Locally Homogeneous Medium," *Radio-Sci.*, Vol. 4, p. 295, 1969.
34. Ivanoff, A., "Optical Properties of Seawater," *Ann de geophysique*, Vol. 13, No. 1, pp. 22-52, 1957.
35. Ivanoff, A., "Polarization Measurements in the Sea," *Symposium on Optical Aspects of Oceanography*, Academic Press, Copenhagen, 1972, to be published.
36. Ivanoff, A., *Lecture Series*, No. 61.
37. Ivanof, A. P. and I. D. Sherbaf, "Optical Conditions in a Turbid Medium Illuminated by a Narrow Light Beam," *Optics and Spectroscopy*, pp. 391-394, 1963.
38. Jerlov, N. G., *Optical Oceanography*, Elsevier Press, 194 pages, 1968.
39. Jitts, H. A., A. Morel and Y. Saijo, "The Relation of Marine Primary Production and Available Photosynthetic Irradiance," to be published.
40. Kourganoff, V., *Basic Methods in Transfer Problems*, Oxford University Press, 1952 and Dover Publi. Inc., 281 pages, 1963.
41. LeGrand, Y., "Penetration of Daylight into the Sea," *Annales de l'institut Oceanographique*, Vol. 14, pp. 393-436, 1939.
42. Lenoble, J., "Application of the Chandrasekhar Method to Study of Scattered Radiation in Fog and in the Sea," *Revue d'optique*, Vol. 35, No. 1, pp. 1-17, 1956.

43. Lenoble, J., "Theoretical Study of Penetration of Radiation into Natural Scattering Media," *Opt. Acts.*, Vol. 4, No. 1, pp. 1-11, 1957.
44. Lenoble, J., "Application of the Spherical Harmonic Method to the Case of Anisotropic Scattering," *C.R.AL. Sci.*, Vol. 252, pp. 2087-2089, Paris, 1961.
45. Lenoble, J., "An Attempt at Establishment of a General Method for Introduction of the X and Y and Chandrasekhar Functions into the case of Anisotropic Scattering," *C. R. Aca. Sci. de Paris*, Vol. 256, pp. 4638--4640, 1963.
46. Lutomirski, R. F. and H. T. Yura, "Wave Structure Function and Mutual Coherence Function of an Optical Wave in a Turbulent Medium," *J.O.S.A.*, Vol. 61, pp. 482-487, 1971. /1.3.21
47. Makarevich, S. A., A. P. Ivanov and G. K. Illich, "The Structure of a Narrow Light Beam Emerging from a Layer of a Scattering Medium," *Atm. and Ocean Physics*, Vol. 5, pp. 77-83, 1969.
48. Marechal, A. and M. Francon, *Diffraction et structure des images, influence de la coherence de la lumiere* [Image Diffraction and Structure — Influence of Light Coherence], Masson Paris, 204 pages, 1970.
49. Mertens, L., *In-Water Photography*, Wiley Interscience, 387 pages, 1970.
50. Mika, J. R., *Nucl. Sci. Eng.*, Vol. 11, p. 415, 1965.
51. Morel, A., "Relationships Between Angular Coefficients and Total Scattering Coefficient of Light for Seawater," *Cahiers Oceanographiques*, Vol. 20, No. 4, pp. 291-303, 1968.
51b. Morel, A., "Study of the Indicatrix of Light Scattering by Seawater for Various Wavelengths," *U.G.G.I. Proces verbaux*, Assembly Berne, No. 10, pp. 204-205, 1967.
52. Morel, A., "Analysis of Experimental Results Relating to the Scattering of Light by Seawater," *AGARD Conference Proceedings on Electromagnetics of the Sea*, No. 77, No. 30, 1970.
53. Morel, A., *Lecture Series*, No. 61, 1973.
54. Nyfeeler, F., "Study of the Scattering of Light by Seawater — Small Angle Measurements by Means of Conventional Sources and Laser Sources," *AGARD Conference Proceedings No. 17 on Electromagnetics of the Sea*, No. 31, 1970.
55. O'Neill, E., *Introduction of Statistical Optics*, Addison Wesley Publ. Comp. Inc., 179 pages, 1963.
56. Pahor, S., "A New Approach to Half Space Transport Problems," *Nucl. Sci. and Eng.*, Vol. 26, p. 192, 1966.
57. Plass, G. N. and G. W. Kattawar, "Influence of Single Scattering Albedo on Reflected and Transmitted Light from Clouds," *Appl. Opt.*, Vol. 7, p. 361, 1968.
58. Plass, G. N. and G. W. Kattawar, "Monte-Carlo Calculations of Light Scattering from Clouds," *Appl. Opt.*, Vol. 7, p. 415, 1968.
59. Plass, G. N. and G. W. Kattawar, "Influence of Particle Size Distribution on Reflected and Transmitted Light from Clouds," *Appl. Opt.*, Vol. 7, p. 869, 1968.
60. Plass, G. N. and G. W. Kattawar, "Calculations of Reflected and Transmitted Radiance for Earth's Atmosphere," *Appl. Optics*, Vol. 7, p. 1129, 1968.
61. Plass, G. N. and G. W. Kattawar, "Radiation Transfer in an Ocean Atmosphere System," *App. Opt.*, Vol. 8, No. 2, pp. 455-466, 1969.

62. Plass, G. N. and G. W. Kattawar, "Thermal Emission from Haze and Clouds," *App. Opt.*, Vol. 9, No. 2, p. 413, 1970.
63. Plass, G. N. and G. W. Kattawar, "Polarization of the Radiation Reflected and Transmitted by Earth's Atmosphere," *App. Opt.*, Vol. 9, No. 5, p. 1122, 1970.
64. Plass, G. N. and G. W. Kattawar, "Monte Carlo Calculations of Radiative Transfer in the Earth's Atmosphere Ocean System. I. Flux in the Atmosphere and Ocean," *Journ. of Phy. Ocean*, Vol. 2, pp. 139-145, 1972.
65. Plass, G. N. and G. W. Kattawar, "Monte Carlo Calculations of Radiative Transfer in the Earth's Atmosphere Ocean System. II. Radiance in the Atmosphere and Ocean," *Journ. of Phy. Ocean*, Vol. 2, pp. 146-156, 1972.
66. Preisendorfer, R. W., "Directly Observable Quantities for Light Fields in Natural Hydrosols," Scripps Institution of Oceanography, University of California, San Diego, Rep. 58-46, 1958.
67. Preisendorfer, R. W., "The Covariation of the Diffuse Attenuation and Distribution Functions in Plane-Parallel Media," Scripps Institution of Oceanography, University of California, San Diego, Rep. 59-52, 1959.
68. Preisendorfer, R. W., "Theoretical Proof of the Existence of Characteristic /1.3-22
Diffuse Light in Natural Water," *J. Mar. Res.*, Vol. 18, pp. 1-9, 1959.
69. Preisendorfer, R. W., "On the Structure of the Light Field at Shallow Depths in Deep Homogeneous Hydrosols," Scripts Institution of Oceanography, San Diego, California, Rep. No. 305. 1959.
70. Preisendorfer, R. W., "A Model for Radiance Distributions in Natural Hydrosols," Tenth Pacific Science Congress, Honolulu, 1961, University of Hawaii Press, pp. 51-59, edited by J. Tyler, 1964.
71. Preisendorfer, R. W., "Radiative Transfer on Discrete Spaces," *Series of Monograph in Pure and Applied Mathematics*, Vol. 74, 462 pages, Pergamon Press, 1965.
72. Preisendorfer, R. W., "A Survey of Theoretical Hydrologie Optics," *J. Quant. Spectrosc. Radiat. Transfer*, Vol. 8, pp. 355-328, 1968.
73. Prieur, L., "Penetration of Daylight into the Sea," *A.G.A.R.D. Conference Proceedings*, No. 77, No. 33, 1970.
74. Prieur, L. and A. Morel, "Theoretical Study of Asymptotic Conditions: Relationships between Optical Characteristics and Extinction Coefficient Relating to Penetration of Daylight," *Cahiers Oceanographiques*, Vol. 23, No. 1, pp. 35-47, 1971.
75. Prieur, L., to be published.
76. Rozenberg, G. V., "The Statistical-Electrodynamic Content of Photometry Quantities and of the Basic Concepts of Radiation Transfer Theory," *Opt. and Spectroscopy*, Vol. 28, pp. 210-213, 1970.
77. Romanova, L. M., "The Solution of the Radiation a Transfer Equation for the Case when the Indicatrix of Scattering Differs from the Spherical One," *Optika i spectroscopya*, Vol. 13, p. 238, 1962.
78. Romanova, L. M., "Solution of the Equation of Radiation Transfer," *Optika i spekt oskopya*, Vol. 13, No. 6, p. 463, 1962.
79. Romanova, L. M., "Small Angle Approximation of a Solution of the Equation for Radiation Transfer, and its Refinement," *Izv. Akad. Nauk. Ser. Geophy.*, Vol. 8, pp. 1108-1112, 1962.
80. Romanova, L. M., "The Radiation Field in Plane Layer of a Turbid Medium with Strongly Anisotropic Scattering," *Optika i Spektroskopya*, Vol. 14, No. 2, pp. 262-269, 1963.

81. Schellenberger, G., "Calculation of the Scattering Function of Natural Water — Underwater Radiation Distribution," *Gerl. Beitr. Geophys.*, Vol. 76, No. 1, pp. 69-82, 1967.
82. Smith, R. C., "The Optical Characterization of Natural Waters by Means of an Extinction Coefficient," *Limn and Ocean*, Vol. 13, No. 3, pp. 423-429, 1968.
83. Smith, R. C., "An Underwater Spectral Irradiance Collector," *J. Mar. Res.*, Vol. 27, pp. 341-351, 1969.
84. Smith, R. C., *App. Opt.*, Vol. 9, pp. 2015-2022, 1970.
85. Sobolev, V. V., *A Treatise on Radiative Transfer*, D. Van Nostrand, Princeton, N.5., 1963.
86. Sugimori, Y and T. Hasehi, "Estimation of Underwater Scattering Radiance up to the Third Order," *J. of Ocean Soc. of Japan*, Vol. 27, No. 2, pp. 73-80, 1971.
87. Tatarskii, V. I., *Wave Propagation in a Turbulent Medium*, Dover Publ., 285 pages, 1967 and McGraw-Hill, NM, 1961.
88. Tatarskii, V. I., *The Effects of the Turbulent Medium on Wave Propagation*, National Science Foundation, Washington, D.C., TT 6850464, 1971.
89. Timofeyeva, V. A., "Similarity Criteria of Turbid Media in Optics," *Izv. Akad Nauk SSSR*, No. 4, pp. 847-850, 1968.
90. Timofeyeva, V. A., "The Diffusion Reflection Coefficient and its Relation to the Optical Parameters of Turbid Media," *Atm. and Ocean Physics*, Vol. 7, No. 6, pp. 688-701, 1971.
91. Timofeyeva, V. A., "Optical Characteristics of Turbid Media of the Seawater Type," *Atm. and Ocean. Physics*, Vol. 7, No. 12, pp. 1326-1329, 1971. /1.3.23
92. Timofeyeva, V. A. and F. J. Gorobets, "Relationships Between Extinction and Attenuation Coefficients," *Bull (Izv) Acad. Sci. USSSR Geophy. Series*, Vol. 3, pp. 291-296, 1967.
93. Timofeyeva, V. A. and L. A. Kovchnikova, "The Problem of Instrumental Determination of the Attenuation Coefficient of Turbid Media," *Atm. and Ocean Physics*, Vol. 2, No. 3, 1966.
94. Timofeyeva, V. A. and V. K. Solomonov, "The Stationary Brightness Distribution Diagram in Turbid Media Such as Seawater," *Atm. and Ocean Physics*, Vol. 6, No. 6, pp. 611-616, 1970.
95. Tyler, J. E., "Program of Research in Optical Oceanography at Scripps Institution of Oceanography," *A.G.A.R.D. Conference Proceedings*, No. 77, No. 43, 1970.
96. Wang, M. C. and E. Guth, "On the Theory of Multiple Scattering Particularly of Charged Particles," *Phys. Rev.*, Vol. 84, No. 6, pp. 1092-1111, 1951.
97. Wells, W. H., "Loss of Resolution in Water as a Result of Multiple Small Angle Scattering," *J.O.S.A.*, Vol. 59, No. 6, p. 686, 1969.
98. Wells, W. H., *Lecture Series*, No. 61.
99. Williams, J., "Optical Properties of the Sea," *United States Naval Institute Series in Oceanography*, 123 pages.
100. Whitman, A. M. and M. J. Beran, "Beam Spread of Laser Light Propagating in a Random Medium," *J.O.S.A.*, Vol. 60, No. 12, pp. 595-1602, 1970.
101. Wolf, D. A., "Saturation of Irradiance Fluctuations Due to Turbulent Atmosphere," *J.O.S.A.*, Vol. 58, p. 461, 1968.
102. Yura, H. T., "Small Angle Scattering of Light by Ocean Water," *Appl. Optics*, Vol. 10, No. 1, pp. 114-118, 1971.

103. Yura, H. T., "First and Second Moment of Wave in Random Medium," *J.O.S.A.*, Vol. 62, pp. 889-892, 1972.
104. Zaneveld, J. R. and J. Beardsley, "Modulation Transfer Function of Seawater," *J.O.S.A.*, Vol. 59, No. 4, pp. 378-380, 1969.
105. Zaneveld, J. R., R. Hodgson and G. F. Beardsley, "Image Degradation over Seawater Paths A Review," *A.G.A.R.D. Conference Proceedings on Electromagnetics of the Sea*, No. 77, No. 41, 1970.
106. Zaneveld, J. R. and H. Pak, "Some Aspects of the Axially Symmetric Submarine Daylight Field," *J. of Geoph. Research*, Vol. 20, pp. 2677-2680, 1972.
107. Zege, E. P., "The Light Field in Duf Layers of a Scattering and Absorbing Medium," *Atm. and Ocean Physics*, Vol. 7, No. 2, pp. 121-132, 1971.

Translated for the National Aeronautics and Space Administration under Contract No. NASw-2485 by Techtran Corporation, P.O. Box 729, Glen Burnie, Maryland, 21061; translator, William L. Hutcheson.